Chaos, Duality and Topology

UIUC Nov 2017

Part 1: Gapless SPTs

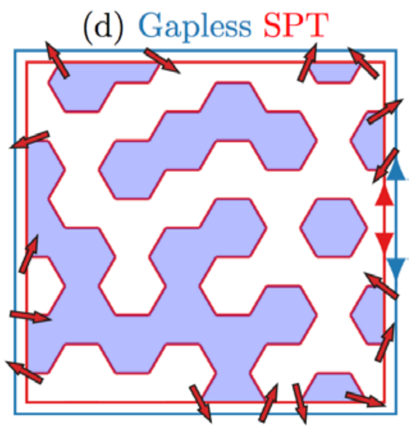
With Daniel Parker and Romain Vasseur

arXiv:1705.01557

PHYSICAL REVIEW X **XX**, 000000 (XXXX)

Gapless Symmetry-Protected Topological Order

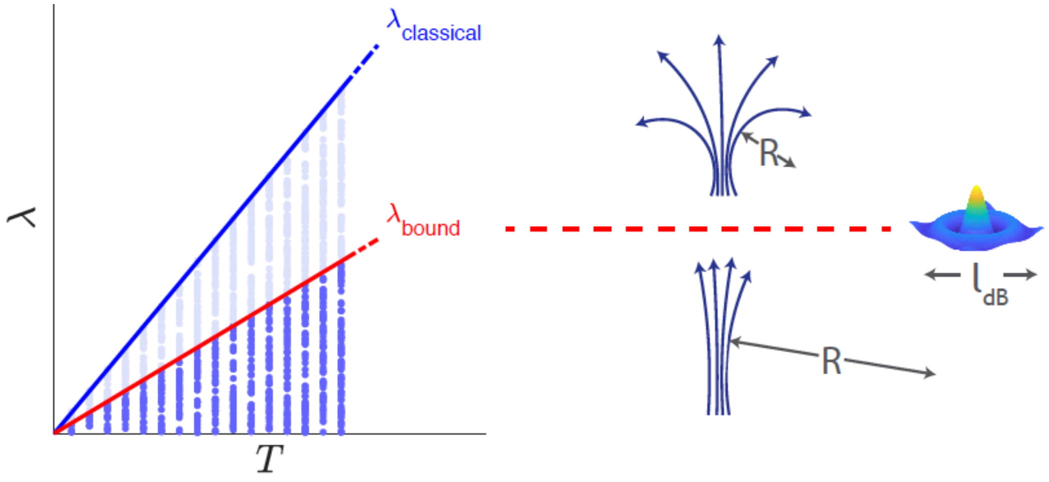
Thomas Scaffidi,[1,*] Daniel E. Parker,[1,†] and Romain Vasseur[1,2,3,‡]



Part 2: Quantum-Classical Correspondence for Maximally Chaotic Systems

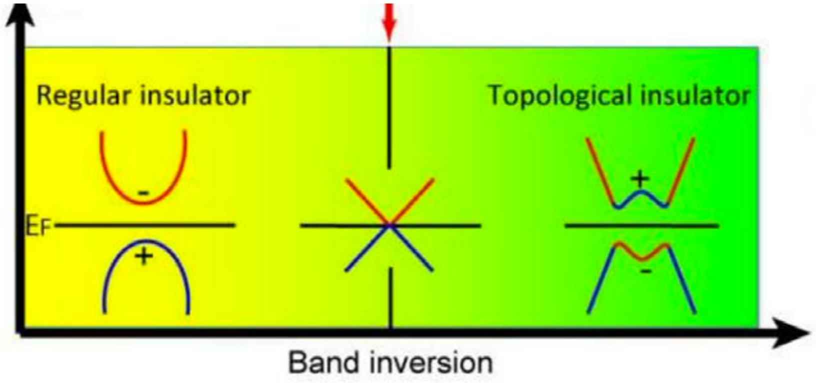
With Ehud Altman

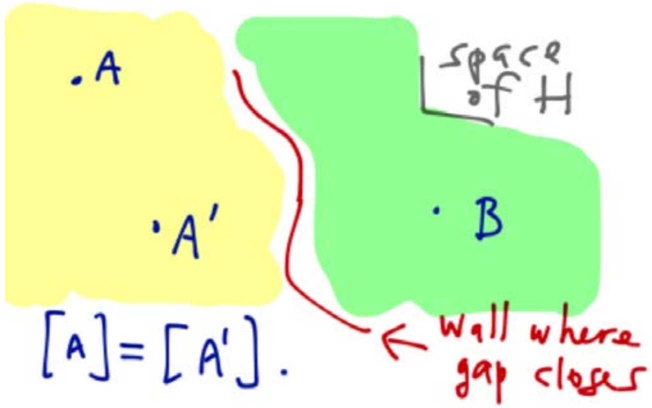
Soon to appear on arXiv



Symmetry-protected topological phases

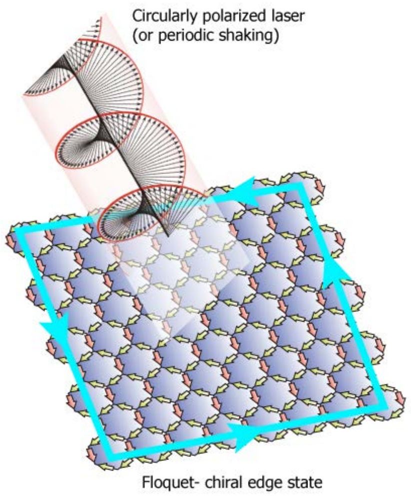
- Gapped
- Equivalence class: two states are in the same phase if one can interpolate between their Hamiltonians without closing the gap and without breaking the symmetry
- Each class is characterized by quantized topological invariants that can only jump at a quantum phase transition
- Protected gapless edge states

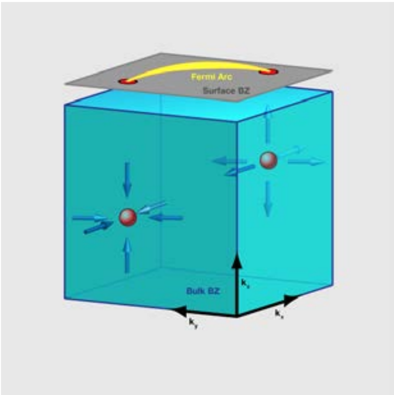


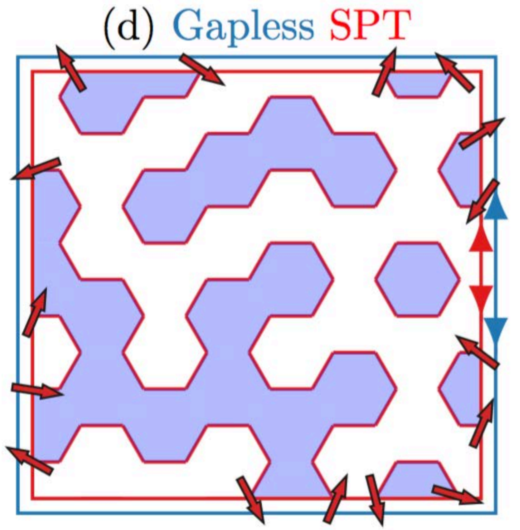


SPTs – Can we go beyond gapped, T=0 phases?

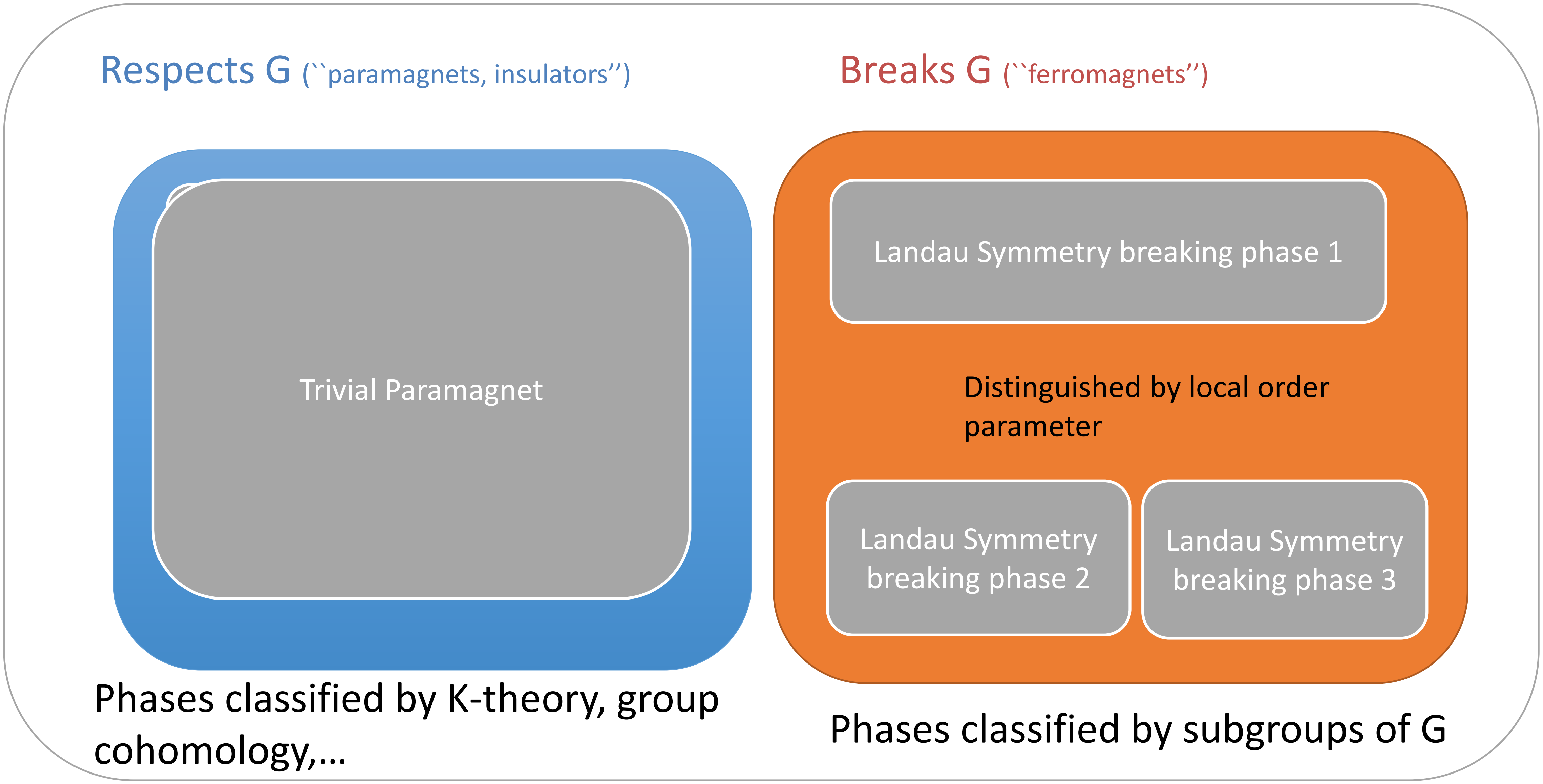
- Non-equilibrium
  - Static MBL
  - Floquet MBL
  - Prethermal systems
- Gapless (free fermions)
  - Weyl semi-metals
- **Gapless (strongly interacting)**
  - Today's topic



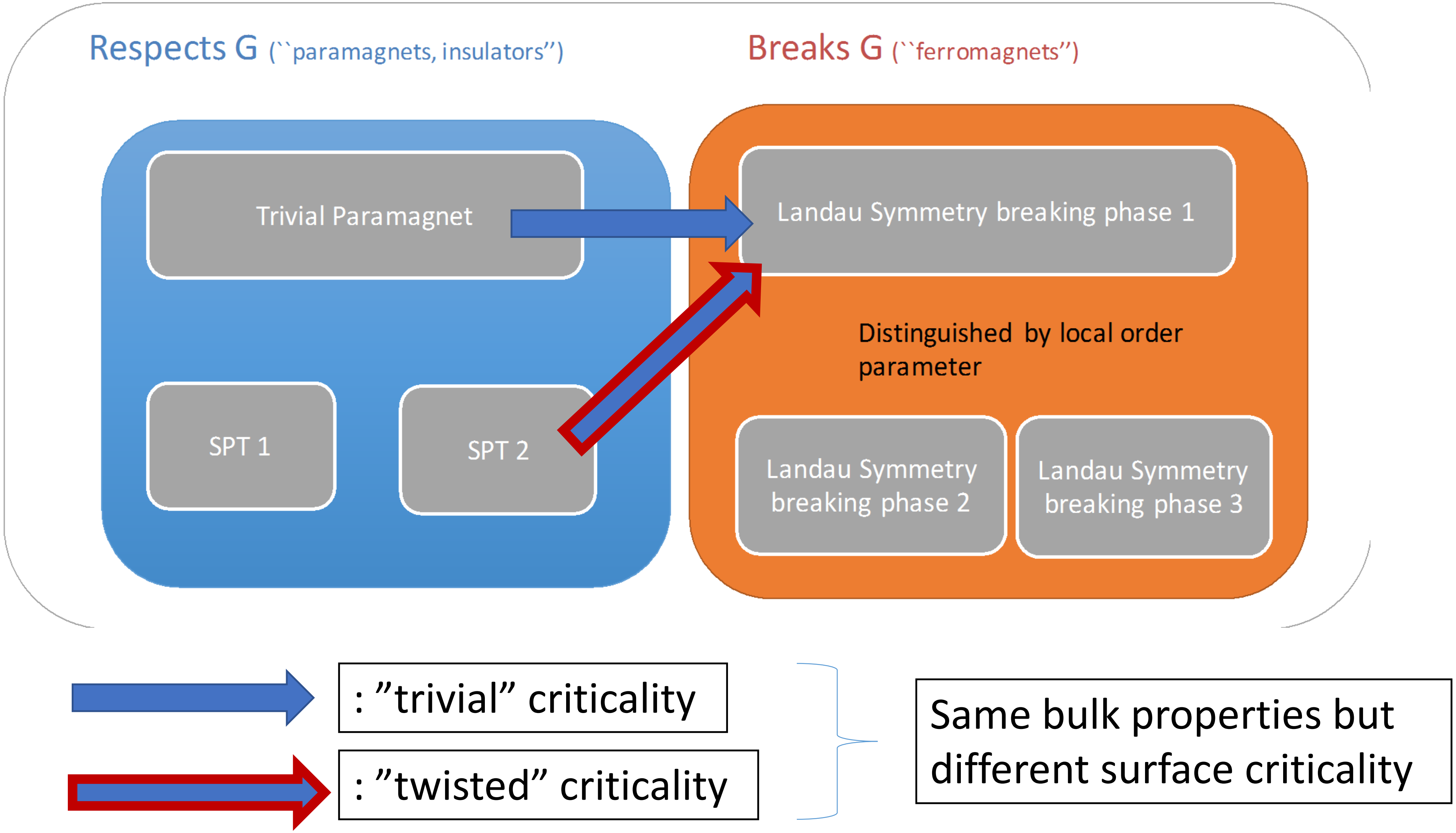




Phase diagram for given symmetry G

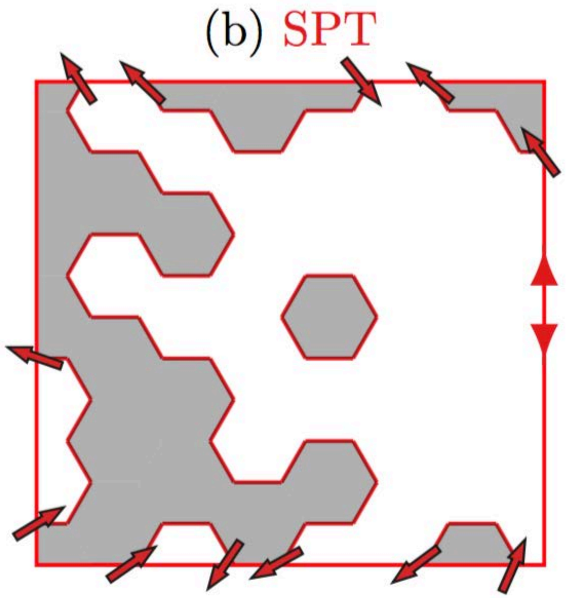


- Starting from an SPT, one can obtain a “twisted” critical point by tuning a subset of the degrees of freedom to criticality



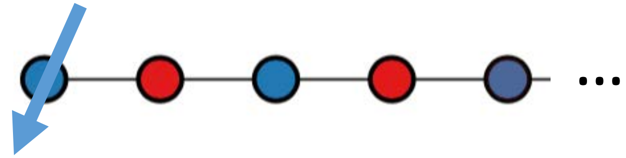
Decorated domain wall picture of SPTs

- SPT with Symmetry Z_2 x G in D dim:
  - Domain walls (or membranes) of Z_2 carry (D-1)-dim SPTs protected by G
- Starting from a 1D SPT protected by Z_2 x Z_2, one gets
  - 2D SPT protected by Z_2 x $(Z_2)^2$
  - 3D SPT protected by Z_2 x $(Z_2)^3$
  - …



- 1D Z_2 x Z_2 (also called cluster state, analogous to Haldane spin-1 chain):

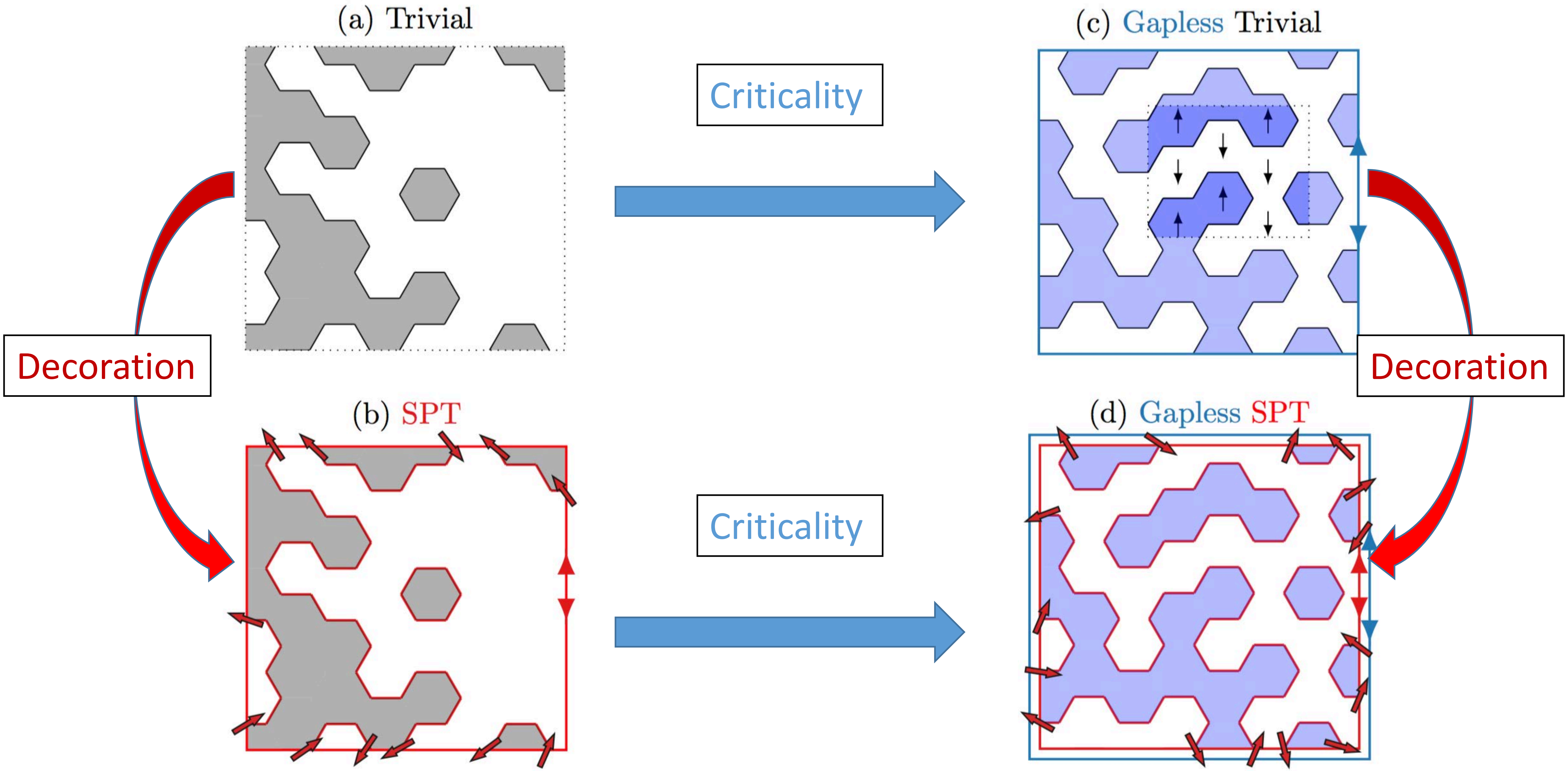
$$H = -\sigma^z_{i-1}\sigma^x_i\sigma^z_{i+1}$$



Free spin 1/2 at the edge

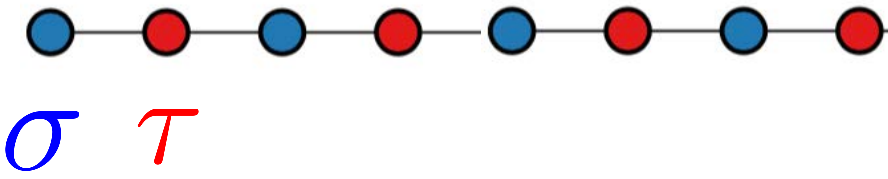
- Free spin 1/2 at the edge generated by σ^z_0 and $\sigma^x_0\sigma^z_1$

General construction for symmetry Z_2 x G

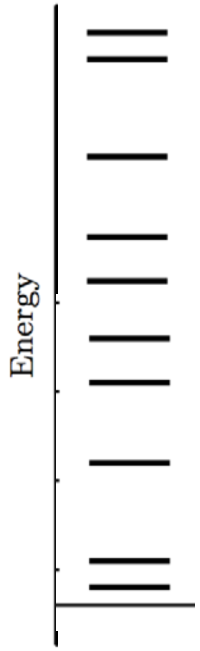


Twisted Ising: (c=1/2)*

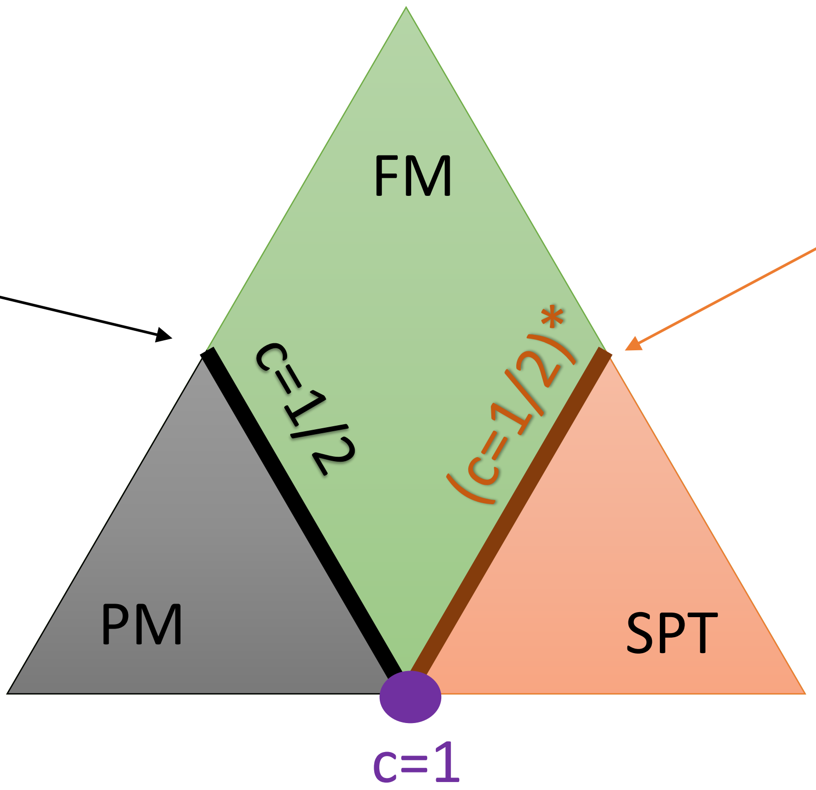
Symmetry: Z2 x Z2



Free boundary



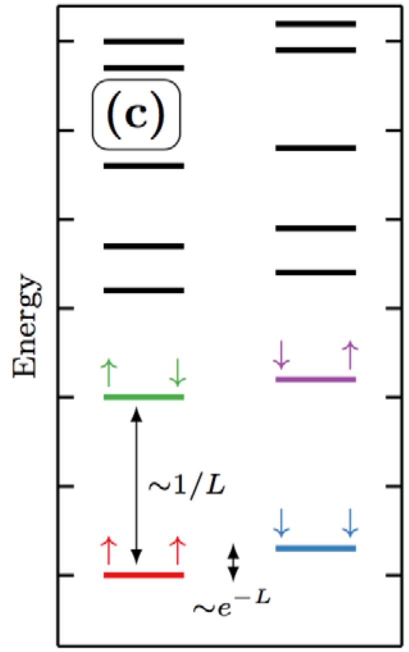
$-J\sigma_i^z\sigma_{i+1}^z$



$H_{\mathrm{PM}} = -h\sigma^x - h\tau^x$

H_{SPT}

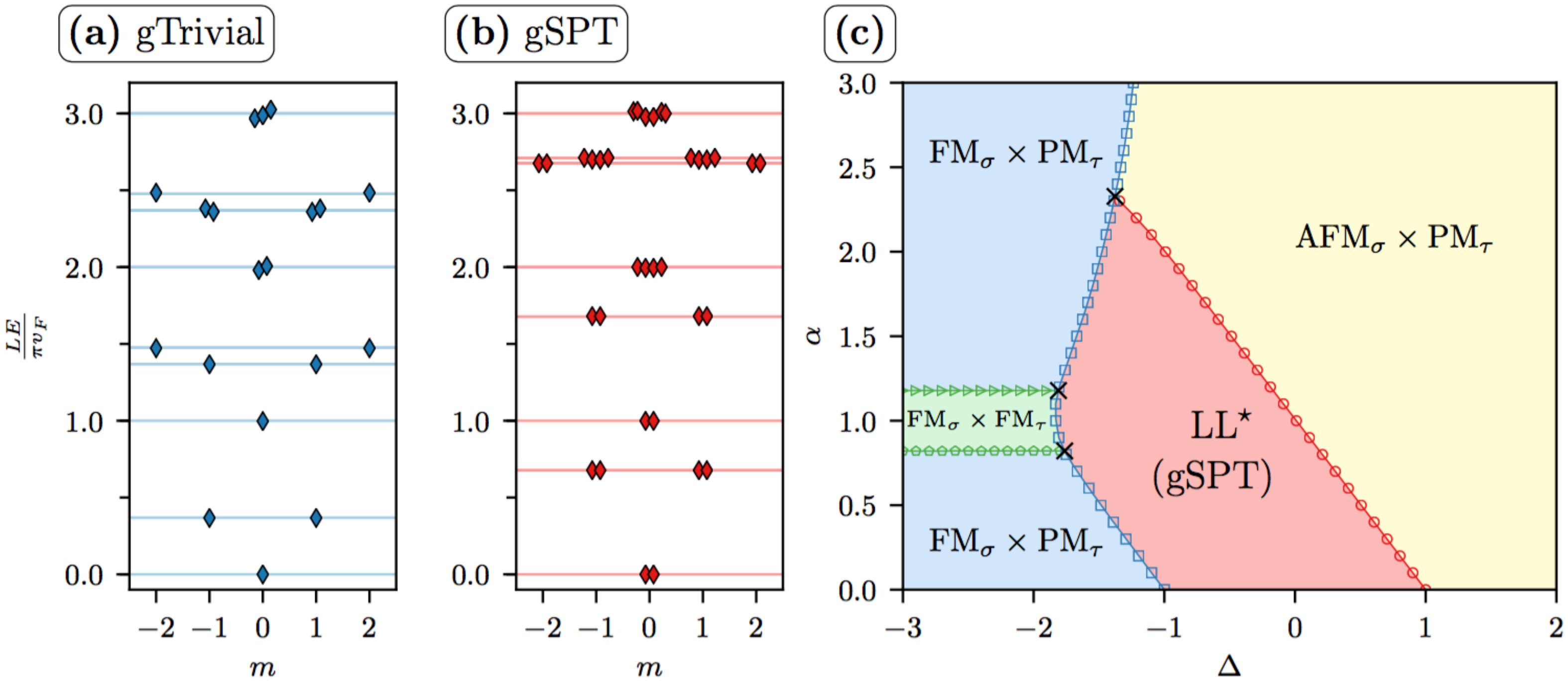
Anomalous boundary



One can also twist critical *phases*:

$$H^{\mathrm{LL}}_{\mathrm{gTrivial}} = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^z \sigma_{i+1}^z + \Delta \sigma_i^y \sigma_{i\text{-}}^y - \sum_i \tau^x_{i-\frac{1}{2}}$$

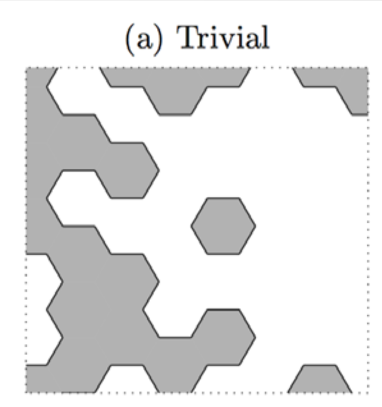
- Topological Luttinger liquids LL*



Result from BCFT: $Z_{\mathrm{LL}^\star} = Z_{0,0} + Z_{0,\pi} + Z_{\pi,0} + Z_{\pi,\pi},$

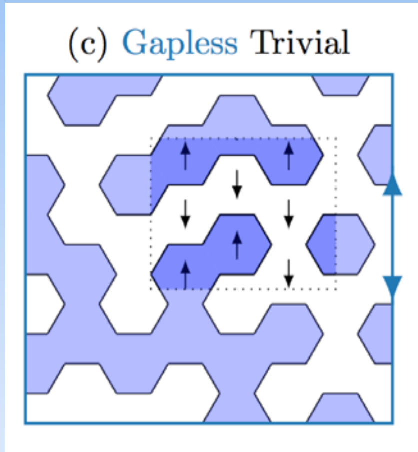
$$Z_{\mathrm{LL}^\star}(g) = \frac{2}{\eta(q)} \sum_{m\in\mathbb{Z}} \left(q^{m^2/g} + q^{(m-\frac{1}{2})^2/g} \right) = 2 Z_{\mathrm{LL}} \left(\frac{1}{4g} \right)$$

2D example: symmetry Z_2 x (Z_2 x Z_2)



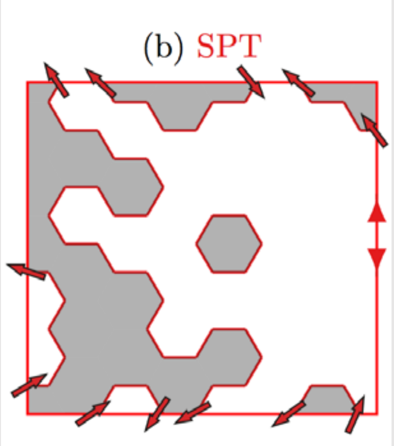
Trivial PM

$$|\Psi\rangle = \sum_{\{DW\}} \bigotimes_{DW} |PM\rangle$$



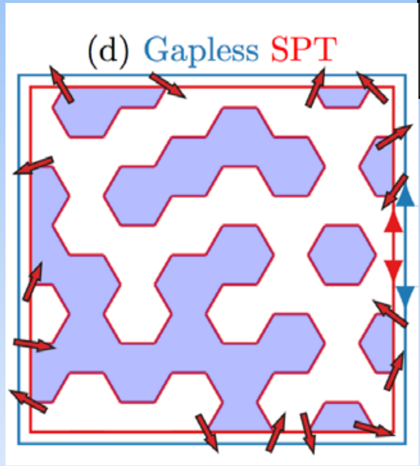
"Trivial" RK point

$$|\Psi\rangle = \overline{\sum_{\{DW\}}} \bigotimes_{DW} |PM\rangle$$



Twisted PM = SPT

$$|\Psi\rangle = \sum_{\{DW\}} \bigotimes_{DW} |AKLT\rangle$$

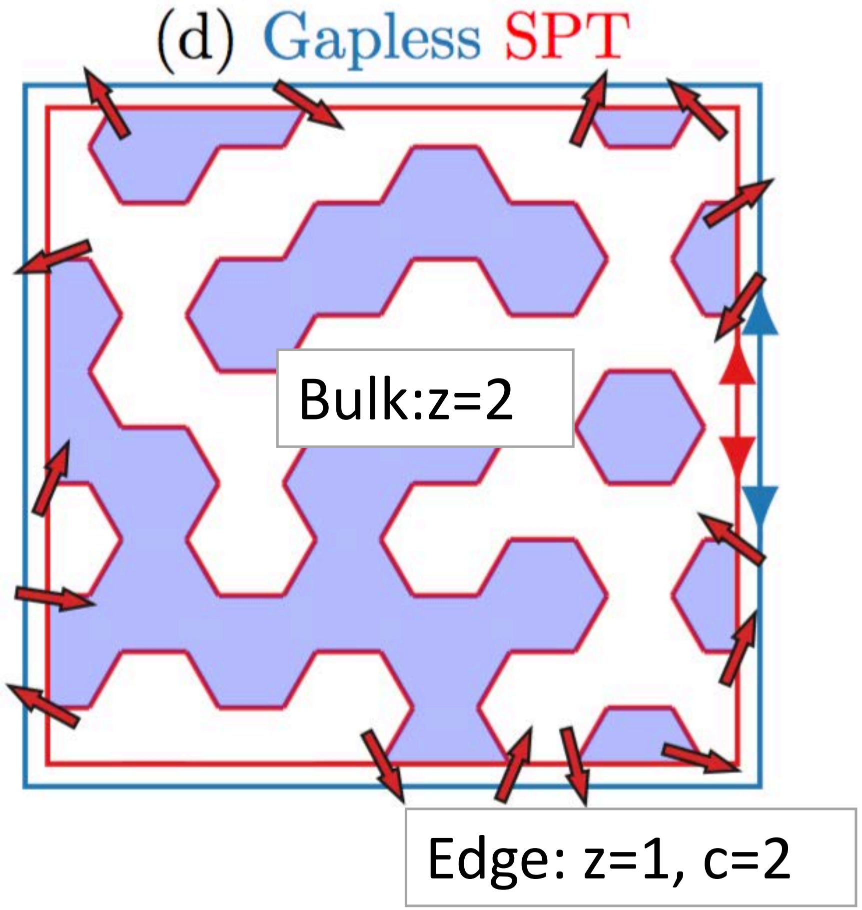


"Twisted" RK point

$$|\Psi\rangle = \overline{\sum_{\{DW\}}} \bigotimes_{DW} |AKLT\rangle$$

"Twisted" RK point

- Evidence for a c=2 edge
  - Field theory
  - Imaginary time-like edge (strange correlators)
  - Entanglement spectrum



- Physical picture: spins are still localized at the edge by the decoration gap, despite the fact that domain walls are critical

The construction leads to many gSPTs:

- 1D
  - Twisted critical Ising
  - Twisted Luttinger liquid
- 2D
  - Twisted Rokhsar-Kivelson models
  - Transition from 2D AKLT to Neel [L. Zhang and F. Wang, PRL 118, 087201]
- … many more to be studied

Part 2:
Classical origin of maximal quantum chaos

With Ehud Altman (Berkeley)

The new results on quantum chaos (quantum bound, dynamics of SYK models, etc) depart from the long history of quantum chaos in the following ways:

1. They pertain to many-body systems
2. Not rooted in a semi-classical limit

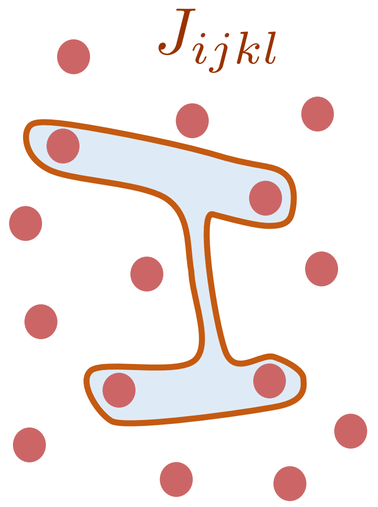
In particular the quantum bound $\lambda_{max} = 2\pi T/\hbar$

does not approach a finite value in the classical limit $\hbar \to 0$

What is then the classical correspondence of a maximally chaotic system?

SYK Model - maximally chaotic

$$\hat{H} = \frac{1}{N^{3/2}} \sum_{ijkl} J_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l \qquad \overline{J_{ijkl}^2} = J^2$$



γ_i - Majorana fermions

- Interesting dynamics beyond saddle point (*O*(*1/N*))
  Kitaev, Maldacena-Stanford, Rosenhaus-Polchinski
  - Solvable model for quantum thermalization and many body chaos
  - Saturates bound on Lyupanov exponent $\lambda_L = 2\pi T$
  - Connection to quantum gravity in AdS_2

First we rewrite the model in terms of the fermion bilinears, which form an so(N) algebra:

$$\hat{L}_{ij} = -\frac{i\hbar}{2}\gamma_i\gamma_j \qquad [\hat{L}_a, \hat{L}_b] = i\hbar f_{abc}\hat{L}_c \qquad a \equiv (i,j)$$

$$a = 1, \ldots, M \quad \text{with} \quad M = N(N-1)/2$$

Classical SYK model may be viewed as a free rotating N-dimensional rigid body with a random inertia tensor:

$$H = \frac{1}{2}\sum_{ab} L_a \mathcal{J}_{ab} L_b \qquad \mathcal{J}_{ab} = J_{ab}/\hbar^2$$

$$\partial_t L_a = f_{abc} L_b \mathcal{J}_{cd} L_d$$

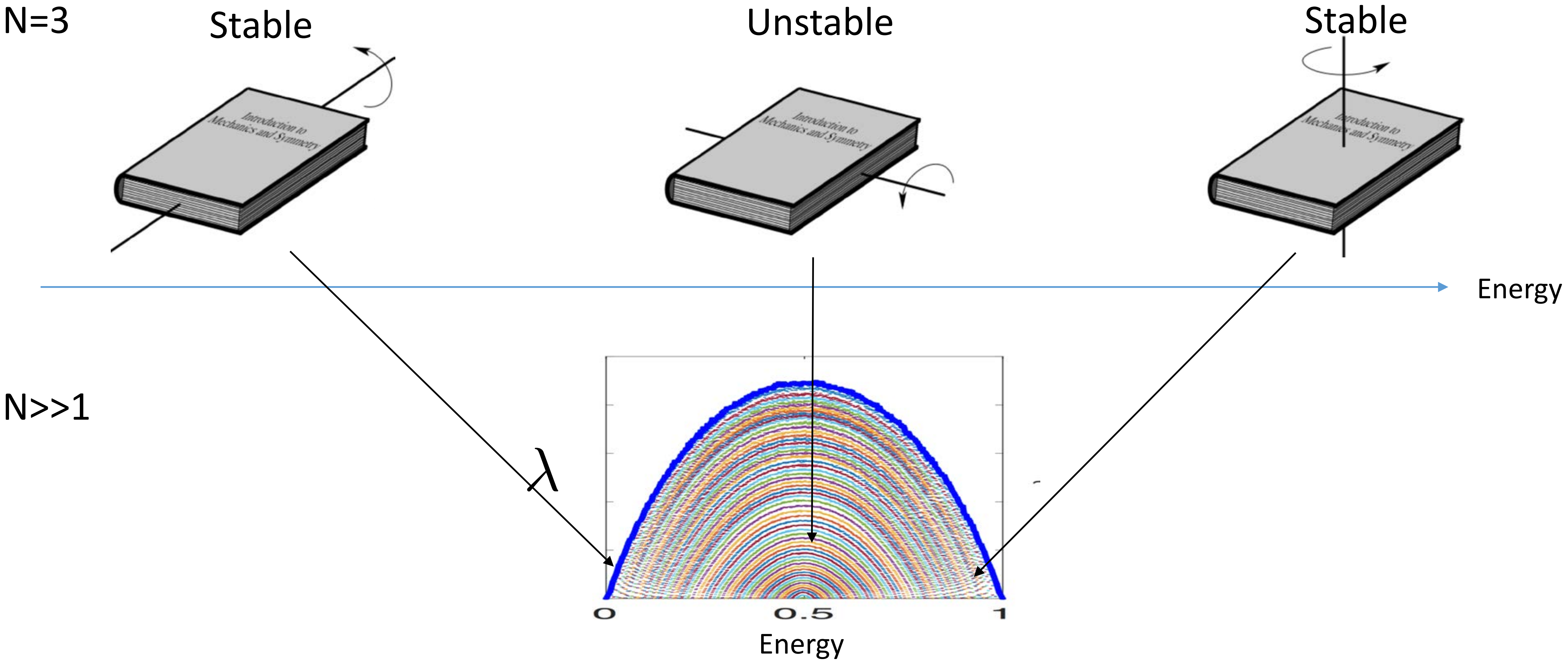
Spin-S spinor representations of SO(N):

Conserved total (angular) momentum: $P^2 = \frac{1}{M}\sum_a L_a^2 \propto S^2$

Intuitive picture of classical SYK

$$H = \frac{1}{2}\sum_{ab} L_a \mathcal{J}_{ab} L_b$$

- Rotation of rigid body with random inertia tensor



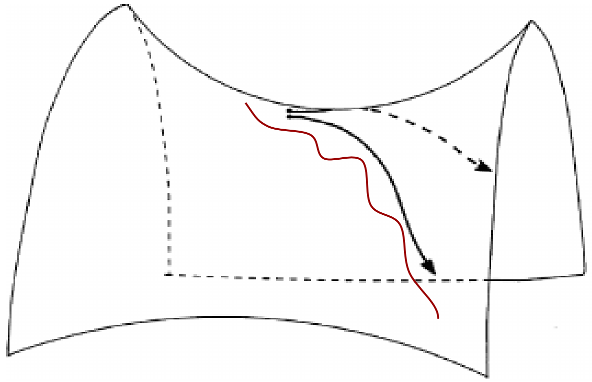
Geometrical picture of classical SYK

$$H = \frac{1}{2}\sum_{ab} L_a \mathcal{J}_{ab} L_b$$

Purely kinetic energy

Free motion on SO(N) manifold with random metric $g_{ab} = \mathcal{J}_{ab}^{-1}$

Random metric => Locally negative curvature => Chaos



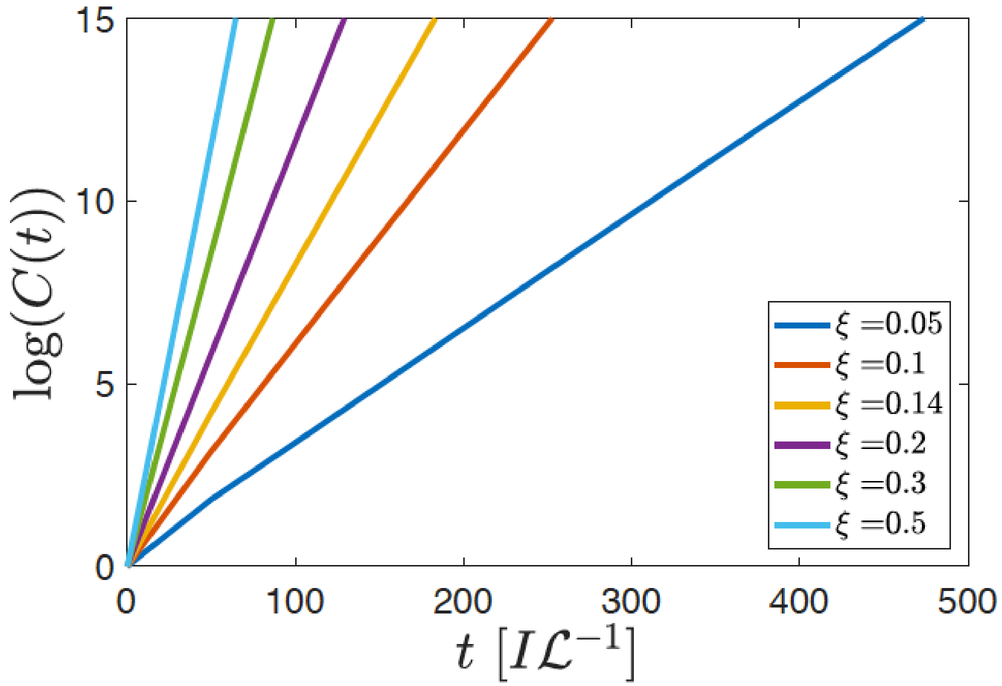
Diagnostic of Chaos

$$\partial_t L_a = f_{abc} L_b \mathcal{J}_{cd} L_d$$

$$C(t) = \frac{1}{M^2} \sum_{a,b} \overline{\left\langle \left(\frac{\partial L_a(t)}{\partial L_b(0)} \right)^2 \right\rangle}$$

Average over initial conditions with constant E and P

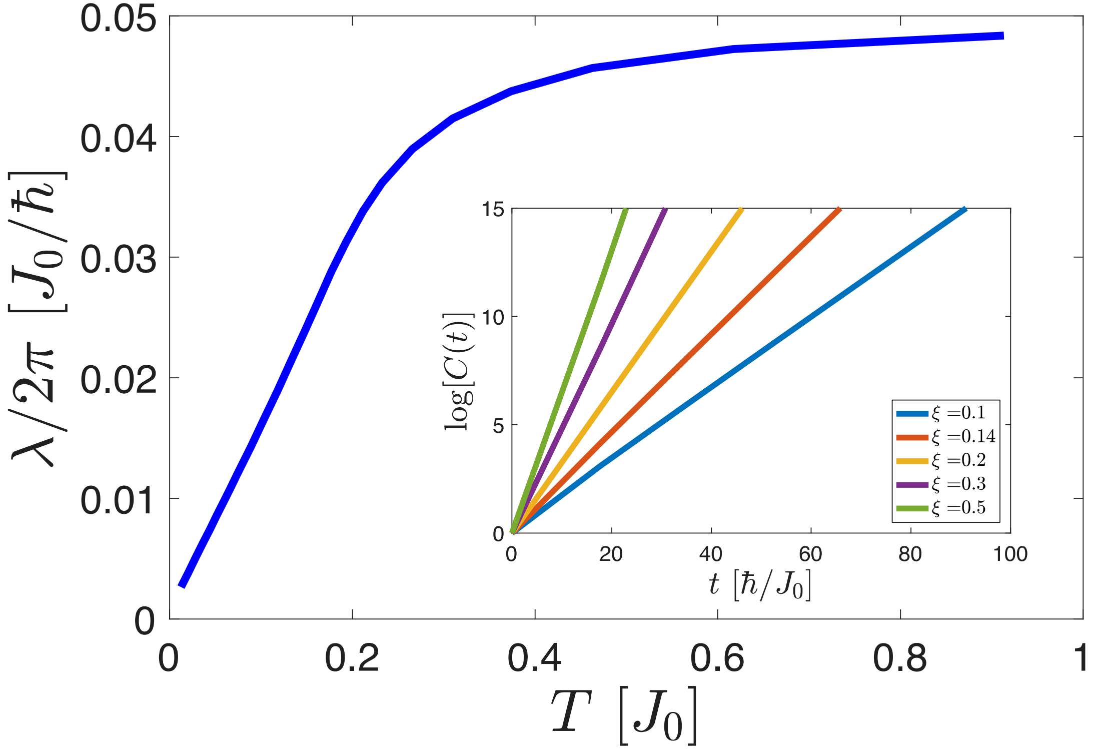
Direct numerical calculation of the Lyapunov exponent:



Exponential growth seen at all energy densities.

➡ Extract Lyapunov.

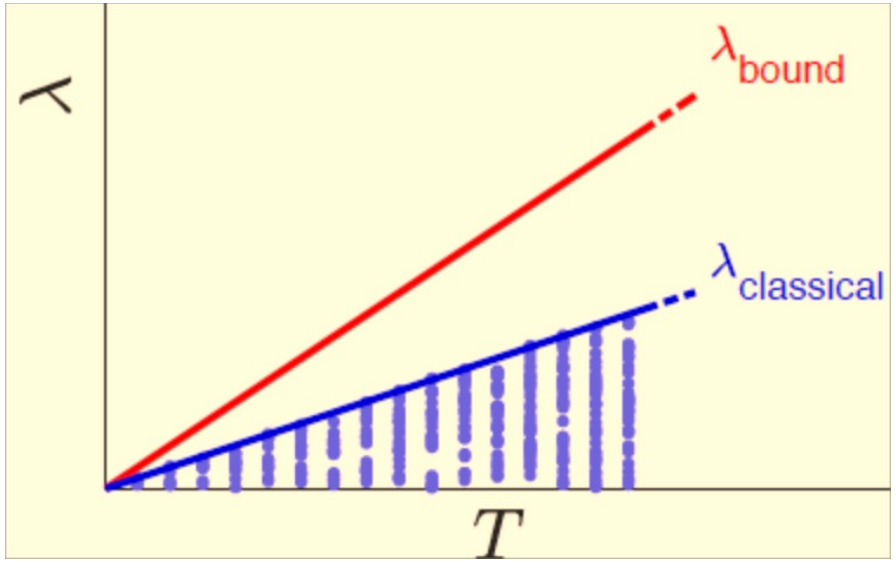
Temperature dependence of λ



The Lyapunov exponent also depends on N and S. We find:

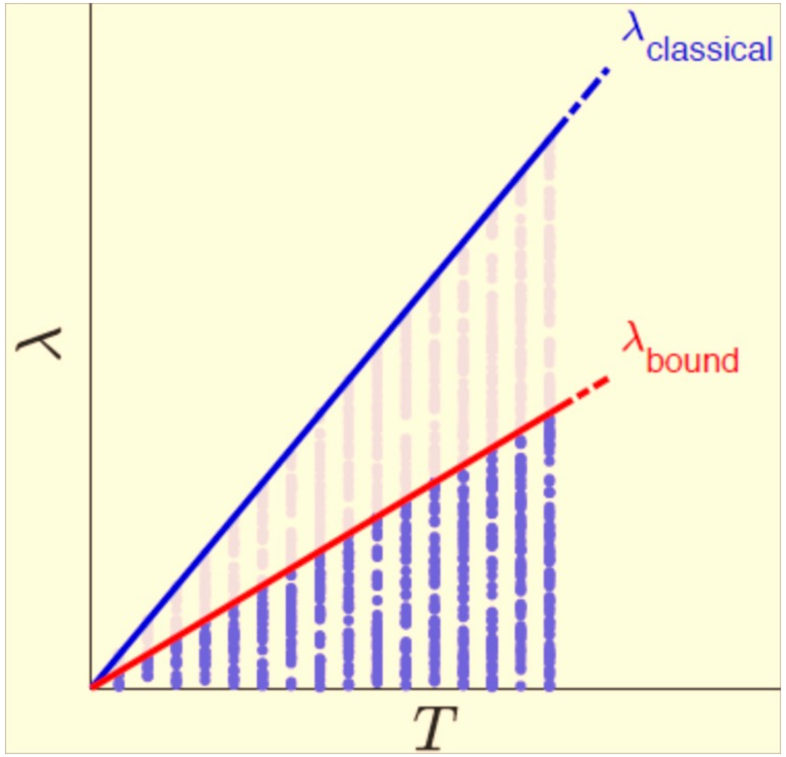
$$\lambda_{cl} \sim \left(\frac{N}{S}\right)\frac{k_B T}{\hbar}$$

$$\lambda_{cl} \sim \left(\frac{N}{S}\right) \frac{k_B T}{\hbar}$$



S>>N:

Semiclassical regime.
Quantum bound satisfied



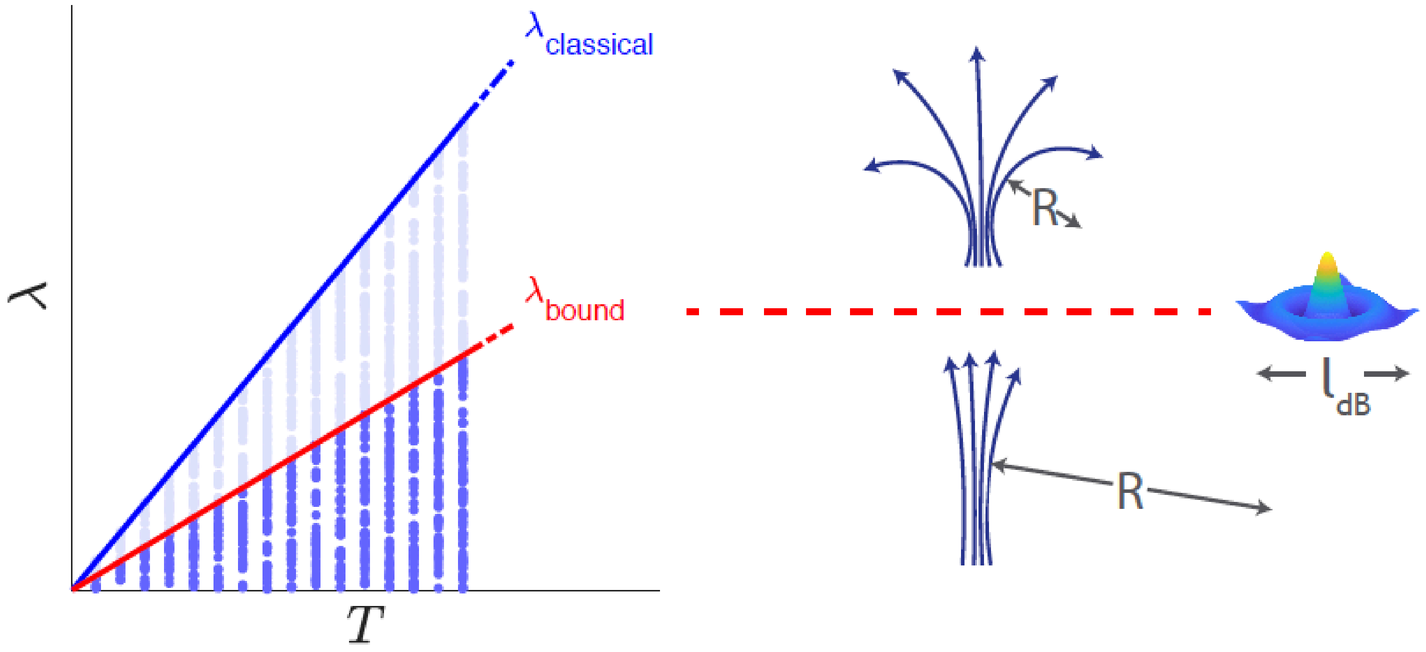
N>>S:

This is the case of interest.
Semi-classics not valid a priori.

Quantum bound exceeded by the classical system! How is it re-established?

The chaotic mobility edge (S<<N)

The classical system has a spectrum of Lyapunov exponents due to the distribution of local curvatures on the SO(N) manifold.



The spectrum is separated into two distinct parts:

Lower = large curvature radii $R>l_{dB}$ ➡ Semi-classical approx. valid

Upper = small curvature radii $R<l_{dB}$ ➡ quantum interference kills chaos

The quantum bound emerges as a crossover from classical to quantum behavior. All chaotic modes are effectively classical!

The semiclassical criterion

Lyapunov exponent related to curvature radius

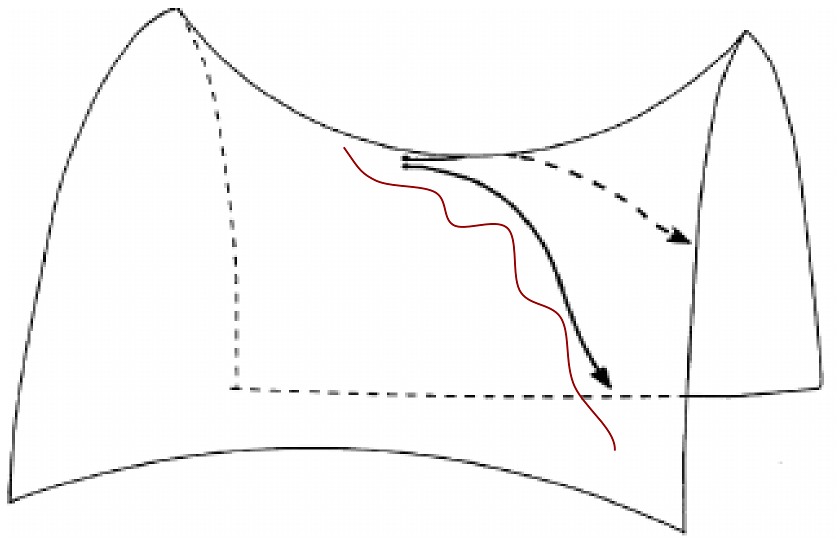
$$\lambda_\nu = \frac{v}{R_\nu}$$

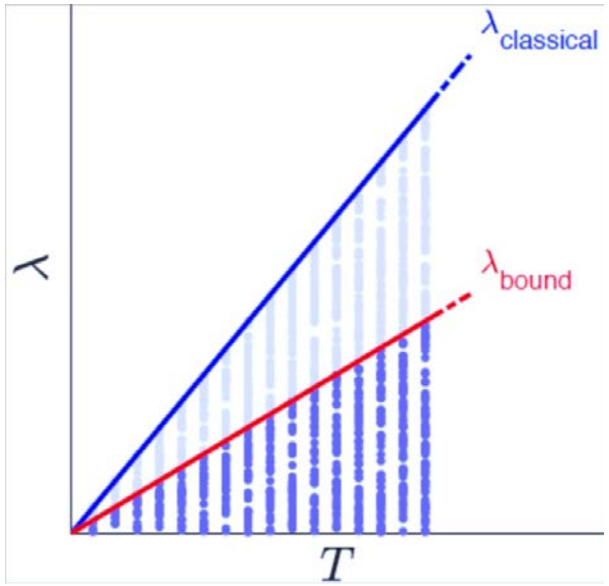
Condition for classical chaos:

$$R_\nu \gg l_{dB} = \hbar / P$$

$$\lambda_\nu < \frac{v}{l_{dB}} = \frac{vP}{\hbar} = \frac{2\epsilon}{\hbar} = \frac{k_B T}{\hbar}$$

Chaotic dynamics is essentially classical,
bounded by the validity of the classical description.





Outlook

- If chaos is classical, can we relate the growth of entanglement entropy to Kolmogorov-Sinai entropy ?
- Is there a phase transition analogous to Anderson localization upon crossing the mobility edge, e.g. by tuning S/N.
- How do these results translate to extended systems not in the large N limit?